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Negative pion capture in closed shell nuclei

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The capture of bound π^- mesons by closed shell nuclei is calculated using the phenomenological Hamiltonian of Eckstein. Numerical calculations are carried out for ^{16}O and the results compared with experiments.

Introduction

The two-nucleon process from nuclei as a result of absorption of a bound π^- meson has recently attracted considerable experimental (Ozaki *et al* 1960, Fedotov 1966, Demidov *et al* 1963, Zaimidarogo 1967, Nordberg *et al* 1968) and theoretical (Ericson 1966, Jibuti & Kopaleishvilli 1964, Cheon 1966, 1967, Nguyen-Truong & Sakamoto, 1967, Koltun & Reitan 1966, 1967, Spector 1964, Eisenberg & Letourneux 1967, Brueckner *et al* 1951, Eckstein, 1963, Divakaran 1965) attention. This has been prompted by the hope that this process may give information about the short-range correlation between the nucleons, since at least two nucleons must participate in the process to conserve energy and momentum. Most calculations proceed by assuming the pion-nuclear interaction to be the sum of the one-body pion nucleon interaction $\frac{f\hbar}{\mu c} \bar{\psi}_N \gamma_S \gamma_\mu \vec{\tau} \psi_N \frac{\partial \vec{\Phi}}{\partial x_\mu}$. The matrix element of the capture process is then calculated assuming a pair-correlation of the two nucleons.

This approach assumes the pion absorption to be a *two-step* process, in which nucleon 1 absorbs the pion then collides with nucleon 2 to share the momentum and since the probability of finding two nucleons close together is higher than finding more than two, the nucleons 1 and 2 leave the region of interaction without any further collisions. The difficulty in this approach is that the two steps are arbitrarily selected out of the many possible way a meson can be absorbed by the two nucleons. Moreover, even if it is allowed that it is the dominant process, we would not know how to calculate the second step since the meson physics involved is extremely complicated. In other words the conventional approach presents an over-simplified picture of the *two-body* character of the meson-nuclear interaction.

Recently Eckstein (1963) following Brueckner *et al* (1951) has proposed an alternative approach. Here the Hamiltonian describing the interaction of the π^- meson with the capturing nucleons is treated as a two-body operator and is constructed phenomenologically. The pion is assumed to be in a relative s-state with respect to the pair of nucleons. The nucleons themselves are also in a relative s-state. The Hamiltonian contains two amplitudes corresponding to captures in $^3S_1$ and $^1S_0$ states (see sec. 2). These amplitudes are determined from the following elementary processes :

$$p + p \to \pi^+ + d,$$
$$p + p \to \pi^0 + p + p. \qquad \ldots(1)$$

Eckstein has used this effective Hamiltonian to calculate the branching ratios of the various modes of capture in ^{4}He, while Divakaran (1965) has calculated the absolute capture rate in ^{3}He. Ericson (1963, 1964) has calculated the imaginary part of the pion-nuclear optical potential using these two-body amplitudes. This calculation has been greatly extended and generalised by Ericson & Ericson (1966) and the optical potential obtained is in very good agreement with experiments (Sens, 1968).

In this paper we calculate the ratio R of the number of neutron pairs emitted to the number of neutron-proton pairs and the angular distribution as a function of the opening single θ between the nucleons using Eckstein's Hamiltonian and shell-model wave functions. The specific examples chosen are closed shell nuclei since the wave function for these nuclei are relatively simple. Numerical calculations are carried out for ^{16}O. We make several appoximations to keep the treatment simple. These and other details of the calculation are described in the next section. It should be mentioned that Ericson (1963) has also calculated the ratio R for carbon and aluminium without, however, making any specific assumption about the nuclear wave-function. In concluding this section we would like to point out that in this approach we need assume a wave-function which describes the behaviour of the nucleons only at large separations. The correlations at short distances are taken care of by the effective Hamiltonian. Therefore, the usual harmonic oscillator wave-functions should be adequate. It is also appropriate to emphasize that since the constants of the Hamiltonian are already determined there are no free parameters in this approach. In the calculations of Jibuti & Kopaleishvilli (1964), Cheon (1966, 1967), Nguyen-Truong & Sakamoto (1967) and Eisenberg & Letourneux (1967) there is usually an adjustable parameter in the correlation function.

Theory

1. *The Hamiltonian,*

The two-body Hamiltonian described in the previous section has the form

$$H_{1i} = \int d^3r_1\, d^3r_2\, \psi^+ (\vec{r}_1)\, \psi^+ (\vec{r}_2)\, M_{12}\, \psi (\vec{r}_1)\, \psi (\vec{r}_2) \qquad \ldots(2)$$

where $\psi^+ (\vec{r})$ and $\psi(\vec{r})$ create and annihilate a nucleon at the point r and M_{12} is an operator involving the momenta, spin and isospin of particles 1 2. The form M_{12} has been extensively discussed by Eckstein (1963) and Divakaran (1965), so we shall merely state it here. :

$$M_{12} = [\, g_0 \{ \tfrac{1}{2} (\vec{\tau}_1 - \vec{\tau}_2) \,.\, \vec{\phi}\,(\vec{r}) \} \tfrac{1}{2} (\vec{\sigma}_1 + \vec{\sigma}_2) \,.\, \vec{k}$$
$$+ g_1 \{ \tfrac{1}{2} (\vec{\tau}_1 + \vec{\tau}_2) \,.\, \vec{\phi}\,(\vec{r}) \} \tfrac{1}{2} (\vec{\sigma}_1 - \vec{\sigma}_2) \,.\, \vec{k}\,]\, T_{12}{}^{\sigma}\, T_{12}{}^{\tau}\, \delta\, (\vec{r}_1 - \vec{r}_2 \qquad \ldots(3)$$

Here $\vec{\phi}$ is the pion field operator. $\vec{\tau}_1$, $\vec{\tau}_2$ and $\vec{\sigma}_1$, $\vec{\sigma}_2$ are the isospin and spin operators respectively, of the two nucleons and $\vec{k}$ is the relative momentum of the outgoing nucleons ; T_{-12}^{σ}, T_{-12}^{τ} are the spin, isospin triplet projection operators. The first term in equation (3) describes the transition from ${}^3S_1$ state of the two nucleons to ${}^3P_1$ state, while the second term describes the transition from ${}^1S_0$ to ${}^3P_0$ state. The amplitudes of these transitions, g_0 and g_1, are determined from the reactions of equation (1).

2. *The Absorption Rate.*

Let $| i >$, $| \psi_f >$, $| \psi_n >$ be, respectively, the inital state consisting of the pion and the ground-state of the target nucleus of A nucleons described by the wave-function $| \psi_i >$, the wave-function of the two outgoing nucleons and the wave-function of the residual nucleons. The matrix-element for pion capture is given by

$$T_{fi} = \sqrt{\tfrac{1}{2} A\,(A-1)} < \psi_n\, \psi_f \mid H_{12} \mid i > \qquad \ldots(4)$$

Here we have neglected the antisymmetry of the emitted nucleons with the residual nucleons. This should be a good approximation considering the high momenta of the outgoing nucleons We also neglect the interaction of the nucleons with the residual nucleons. The absorption rate is given by

$$dW = 2\,\pi\delta\,(E_n + E_f - E_i)\, \tfrac{1}{2} A\,(A-1) \mid < \psi_n\, \psi_f \mid H_{12} \mid i > \mid^2$$
$$\times \frac{\Omega\, d^3k_1}{(2\pi)^3} \frac{\Omega\, d^3k_2}{(2\pi)^3}\, ; \qquad \ldots(5)$$

$\vec{k}_1$ and $\vec{k}_2$ are the momenta of the outgoing nucleons and Ω is the quantization volume. The energies occurring in the δ - function are

$$E_i = m_\pi - B_t$$
$$E_n + E_f = E_1 + E_2 - B_r + E_n + E_r$$

where m_π is the rest mass of the pion ($\hbar = c = 1$). E_1, E_2 and E_r are, respectively, the kinetic energy of the emitted nucleons, and the residual nucleus. B_t and B_r are the binding energies of the target and residual nuclei while E_n is the excitation energy of the latter. We have now to sum over the states n of the residual system. To do this we follow Brueckner *et al* (1951) and invoke the closure approximation, that is, assume that the states of the residual nucleus belong to a narrow range of energy defined by an average

$\bar{E} = < E_n + E_r + B_i - B_r >_{AV} \ll m_\pi$. Carrying out the sum over n in (4) with this assumption we get

$$dW = 2\pi\delta\,(E - \bar{E})\,\frac{\Omega\, d^3k_1}{(2\pi)^3}\,\frac{\Omega\, d^3k_2}{(2\pi)^3}\,u \qquad \ldots(6)$$

where

$$E = E_1 + E_2 - m_\pi$$

and

$$u = \tfrac{1}{2} A\,(A-1) \int dx_1{}'\, dx_2{}' \int dx_1\, dx_2 \int dx_3 \,..\, dx_A \phi_\pi{}^* (x_1)\, \phi_\pi (x_1{}')$$
$$\times\, \psi^*{}_f\,(x_1, x_2)\, M_{12}\, \psi_i\,(x_1, x_2, x_3 \ldots, x_A)\, \psi^*{}_i\,(x_1{}', x_2{}', x_3, \ldots, x_A)$$
$$M^+{}_{1'2'}\, \psi_f\,(x_1{}', x_2{}') \qquad \ldots(7)$$

Here x_i stands for the discrete and the continuous variables and the integration sign includes the summation over the discrete variables. ϕ_π is the wave-function of the bound pion.

We shall now specialize to the case of a closed shell nucleus. We can therefore choose for ψ_i a single determinant constructed with harmonic oscillator functions $\phi_\alpha\,(\vec{r})$ where α stands for (nlm) which specify the orbit and τ, the charge of the particle. Then the two-particle density matric ρ given by

$$\rho\,(x_1{}', x_2{}', x_1, x_2) = \tfrac{1}{2} A\,(A-1) \int dx_3 \ldots dx_A\, \psi_i\,(x_1, x_2, x_3 \ldots x_A)$$
$$\psi^*{}_i\,(x_1{}', x_2{}', x_3, \ \ , x_A)$$
$$= \tfrac{1}{2} \sum_{\alpha\beta} \phi_\alpha{}^*\,(\vec{r}_1{}')\, \phi_\beta{}^*\,(\vec{r}_2{}')\,\{\phi_\alpha\,(\vec{r}_1)\,\phi_\beta\,(\vec{r}_2) - \phi_\alpha\,(\vec{r}_2)\,\phi_\beta\,(\vec{r}_1)\} \qquad \ldots(8)$$

The final-state wave function ψ_f of the outgoing nucleons is given

$$\psi_f(12) = \frac{1}{\Omega} \exp i\,(\vec{K}\,.\,\vec{R} + \vec{k}\,.\,\vec{r})\, \chi_{S_f M_f}\, \xi_{T_f \lambda_f} \qquad \ldots(9)$$

where $\chi_{S_f M_f}$ and $\xi_{T_f \lambda_f}$ are, respectively, the spin and isospin functions of the two particles. Here

$$\vec{K} = \vec{k}_1 + \vec{k}_2, \qquad \vec{k} = \tfrac{1}{2}\,(\vec{k}_1 - \vec{k}_2)$$
$$\vec{R} = \tfrac{1}{2}\,(\vec{r}_1 + \vec{r}_2), \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

Substituting equation (9) in (7) and carrying out the sums over the isospin quantum numbers we obtain for the capture in ${}^3S_1$ and ${}^1S_0$ states of the pair, respectively,

$$u_0 = \frac{1}{2\Omega^2} g_0^2 F_1(\vec{k}, \vec{K}) \quad \ldots(10a)$$

$$u_1 = \frac{1}{2\Omega^2} g_1^2 F_2(\vec{k}, \vec{K}) \quad \ldots(10b)$$

where,

$$F_1(\vec{k}, \vec{K}) = \sum_{\alpha\beta} \int d^3r\, d^3r'\, e^{i\vec{K}\cdot(\vec{r}-\vec{r}')} \phi_\alpha^*(\vec{r}')\, \phi_\beta^*(\vec{r}')\, \phi_\pi^*(\vec{r}')$$
$$\times \{\tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{k}\}\, \phi_\alpha(\vec{r})\, \phi_\beta(\vec{r})\, \phi_\pi(\vec{r}) \quad \ldots(11)$$

and

$$F_2(\vec{k},\vec{K}) = \sum_{\alpha\beta} \int d^3r\, d^3r'\, e^{i\vec{K}.(\vec{r}-\vec{r}')} \phi_\pi^*(\vec{r}')\phi_\beta^*(\vec{r}'\phi_\alpha^*(\vec{r}')$$
$$\times \left(\frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2).\vec{k}\right)\phi_\alpha(\vec{r})\phi_\beta(\vec{r})\phi_\pi(\vec{r}) \quad \ldots(12)$$

The integral over the space coordinates can be carried out by expanding the exponential in spherical harmonics and integrating over the angles. Substituting equations (10a) and (10b) in (6) we obtain the absorption rates,

$$dW_0 = 2\pi\delta(E - \bar{E})g_0^2 F_1(\vec{k}_1\vec{K}) \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \quad . \ (13a)$$

$$dW_1 = 2\pi\delta(E - \bar{E})g_1^2 F_2(\vec{k},\vec{K} \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \quad . \ (13b)$$

The expression for the capture rate is thus given simply as product of three factors : phase-space, the probability of transition of the capturing nucleons from S to P states and the probability of finding them with a total momentum $\vec{K}$. Since we consider the capture of π^- by the two nucleons at zero separation only these two probabilities appear as product.

We now apply the above formulae to ${}^{16}O$. The summations in (11) and (12) range over the levels os and op only. Also for comparing with experimental results we have to integrate the expressions (13a) and (13b) over E_1 and E_2, since they are not measured. Carrying out the summations and integrations we get, for capture from $1S$ orbit

$$d\sigma_0^S = \frac{g_0^2}{2\pi^4} \frac{m_p}{m_\pi}\left(\frac{Z}{a_0}\right)^3\left(4I_0 + \frac{1}{4\nu^2} I_2\right) \quad \ldots(14a)$$

$$d\sigma_1^S = \frac{g_1^2}{2\pi^4} \frac{m_p}{m_\pi}\left(\frac{Z}{a_0}\right)^3\left(4I_0 + \frac{1}{4\nu^2} I_2\right) \quad \ldots(14b)$$

for capture from $2P$ orbit

$$d\sigma_0{}^P = \frac{g_0{}^2}{64\pi^4} \frac{m_p}{m_\pi} \left(\frac{Z}{a_0}\right)^5 \frac{1}{\nu} \left(18I_0 - \frac{2}{\nu} \cdot I_1 + \frac{3}{4\nu^2} \cdot I_2 + \frac{1}{16\nu^3} \cdot I_3'\right) \qquad \ldots(15a)$$

$$d\sigma_1{}^P = \frac{g_1{}^2}{128\pi^4} \frac{m_p}{m_\pi} \left(\frac{Z}{a_0}\right)^5 \frac{1}{\nu} \left(18I_0 - \frac{2}{\nu} \cdot I_1 + \frac{3}{4\nu^2} \cdot I_2 + \frac{1}{16\nu^3} \cdot I_3\right) \qquad \ldots(15b)$$

where

$$I_n = \int k_1{}^2\, dk_2 \int k_2{}^2\, dk_2\, \delta(a^2 - k_1{}^2 - k_2{}^2)\, k^2\, K^{2n} \exp\left(-\frac{K^2}{2\nu}\right)$$

and

$$a^2 = 2m_p\, \bar{E}$$

Here $\nu = 0.323\, f_m{}^{-2}$, the oscillator constant as determined from electron scattering. The wave-functions of the bound pions have been approximated as follows

$$\phi_\pi^{2S}(r) \approx \phi_\pi^{2S}(0) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2}$$

$$\phi_{2\pi}{}^{2P}(r) \approx \left(\frac{Z}{2a_0}\right)^{3/2} \frac{Z}{a_0\sqrt{3}}\, r\, Y_{1m}(\theta, \phi)$$

where a_0 is the Bohr radious of the pionic atom.

Results and discussion

(i) We first calculate the total capture rate. To do this we have to estimate the value of $\bar{E}$ which is the average of the difference in energy between the outgoing nucleons and the rest mass of the pion. The difference of the binding energies $B_r - B_i$ is 23 MeV. The recoil energy varies from 0 to 15 MeV depending on the angle θ. Therefore $\bar{E}$ will depend on θ. The value of $\bar{E}$ has been fixed at 35 MeV in our calculations. We have verified that our results are not very sensitive to small changes in $\bar{E}$.

The total capture rate is obtained by integrating the expressions (14a) (15b) over θ. The values of the constants $g_0{}^2$ and $g_1{}^2$ are

$$g_0{}^2 = 0.60 \pm 0.05\, f_m{}^8$$
$$g_1{}^2 = 0.29 \pm 0.15\, f_m{}^8$$

The value of $g_0{}^2$ differs from that of Eckstein (1963) because we have used the more recent experimental results of Rose (1967) for $p + p \rightarrow \pi^+ + d$ cross section. Using these values for the amplitudes one obtains

$$\sigma_{tot.}{}^{1S} = (1.42 \pm 0.32) \times 10^{18}\ \text{sec}^{-1}.$$

The experimental value as measured by Backenstoss *et al* (1967) is $(11{\cdot}1 \pm 0{\cdot}1) \times 10^{18}$ sec^{-1}.

The calculated absorption rate from $2P$ orbit is $\sigma_{tot}^{2P} = 2 \times 10^{15}$ sec^{-1}. This is $3 - 4$ times larger than the $2P \rightarrow 1S$ radiative transition rate. This is in accord with the experimental observation that most absorptions take place from $2P$ orbit.

(ii) The angular distribution for absorption from $2P$ orbit (since most absorptions take place from this orbit) is plotted in figure 1. The experimental points and the theoretical curve are normalized to unity at $\theta = 180°$. With this normalization the theoretical curve does not distinguish between the emission of (nn) and (np) pairs [see equations (15*a*) and (15*b*)].

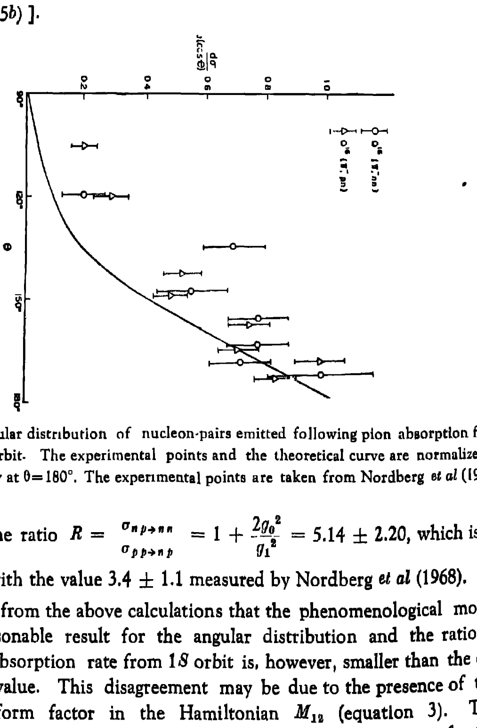

Figure 1. Angular distribution of nucleon-pairs emitted following pion absorption from $2P$ orbit. The experimental points and the theoretical curve are normalized to unity at $\theta = 180°$. The experimental points are taken from Nordberg *et al* (1968)

(iii) The ratio $R = \frac{\sigma_{np \rightarrow nn}}{\sigma_{pp \rightarrow np}} = 1 + \frac{2g_0^2}{g_1^2} = 5.14 \pm 2.20$, which is in agreement with the value 3.4 ± 1.1 measured by Nordberg *et al* (1968).

We see from the above calculations that the phenomenological model gives a reasonable result for the angular distribution and the ratio R. The total absorption rate from $1S$ orbit is, however, smaller than the experimental value. This disagreement may be due to the presence of the δ-function form factor in the Hamiltonian M_{12} (equation 3). The δ-function allows only relative angular momemtum $l = 0$ states for the nucleons, whereas, in ^{16}O, l can have values upto 2. It should be pointed out that the neglect of absorption of p-wave pions is not likely to affect our results, since it has been shown by Ericson & Ericson (1966) that the $1S$ level width is not affected by the p-wave pion. Lastly, the assumption of a single determinant of harmonic oscillator wave-functions for ^{16}O is not a limitation as far as this calculation is concerned. Because it describes the behaviour of nucleons at large distances correctly, as is known from electron scattering data (Ehrenberg *et al* 1959). A modification of the form factor in the effective Hamiltonian may therefore be necessary to achieve a better agreement of the absorption rate with the experimental value.

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